King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Information and Computer Science Department
ICS 253: Discrete Structure I
Fall Semester 2021-2022 (211)
Final Exam, Wednesday 22 December 2021
Duration: 100 Minutes

«Serial» «Name» «M_00Id»
Section 01

This Exam has 2 parts. Part I has 3 problem-solving questions and Part II has 35 Multiple-choice questions (MCQ).

Important Instructions:
1. Make sure you have 11 pages.
2. Your ID and Name are written at the bottom of each page.
3. The exam is closed book and closed notes.
4. All types of calculators, pagers, or mobile phones are **NOT** allowed during the examination.
5. Answer all questions and make sure your answers are clear and readable and only in the indicated places.

Question	Maximum	Earned	Remarks
Part I	**30**		
Part II	**70**		
Total	**100**		

Question 1: [10 points] Solving Recurrence Relations
What is the solution of the recurrence relation $a_n = 4a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 0$ and $a_1 = 4$?

Question 2: [10 points] Sets and Mathematical Induction
Prove that if $A_1, A_2, \cdots, A_n$ and B are sets, then $(A_1 \cup A_2 \cup \cdots \cup A_n) \cap B = (A_1 \cap B) \cup (A_2 \cap B) \cup \cdots \cup (A_n \cap B)$

Question 3: [10 points] Introduction to Proofs
Use the logical equivalence laws to show that $r \rightarrow [\neg(r \rightarrow s) \vee s]$ is a tautology. Clearly show all steps. (Do not use truth tables)

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Part II. [70 points]: MCQ (2 points each)

Write X for your choice to each question after answering all MCQ questions

(Grading will be based on the choices in this table only)

A		Your choice for each question				
		a	b	c	d	e
	Example			X		
Question Number	1					
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Question 1: [2 points] Probability Theory
What is the probability that the numbers 5, 11, and 22 are drawn in that order from a bin containing 40 balls labeled with the numbers 1, 2, . . ., 40 if the ball selected is returned to the bin before the next ball is selected?

a) $\frac{1}{40} \times \frac{1}{39} \times \frac{1}{38}$
b) $\frac{5}{40} \times \frac{11}{40} \times \frac{22}{40}$
c) $\frac{5}{40} + \frac{11}{40} + \frac{22}{40}$
d) $\frac{1}{40^3}$
e) None of the other answers is Correct.

Question 2: [2 points] Solving Linear Recurrence Relations
Given the following recurrence relations:

1. $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$
2. $a_n = 2na_{n-1} + a_{n-2}$
3. $a_n = a_{n-1} + a_{n-4}$
4. $a_n = a^2_{n-1} + a_{n-2}$
5. $a_n = a_{n-1} + 2$
6. $a_n = a_{n-2}$
7. $a_n = a_{n-1} + n$

Select all linear homogeneous recurrence relations with constant coefficients from the following:

a) 2, 3, and 5 only
b) 1, and 2 only
c) 5 and 7 only
d) 1, 3, and 6 only
e) None of the other answers is correct.

Question 3: [2 points] Predicates and Quantifiers
If F(p) is "Printer p is out of service," B(p) is "Printer p is busy," L(j) is "Print job j is lost," and Q(j) is "Print job j is queued." Then,
$\forall p \exists j ((\neg F(p) \vee \neg B(p)) \rightarrow (L(j) \vee Q(j)))$ is equivalent to

a) There exists a job for all printers such that if the printer is working or is not busy, then the job is either lost or queued.
b) For each printer that is working or is not busy, a job is either lost or queued.
c) All printers that are out of service or busy have all their jobs either lost or queued.
d) There exists a printer out of service with all its jobs not lost and not queued.
e) There exists a printer that it is out of service or is busy and all its jobs are either lost or queued.

Question 4: [2 points] Recurrence Relations
Given the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$
where $a_0 = 2$, $a_1 = 5$, and $a_2 = 15$
The characteristic equation of the above recurrence relation is:

a) $r^3 = 6r^2 + 11r - 6$
b) $r^3 = 6r^2 - 11r + 6$
c) $r^3 + 6r^2 - 11r + 6 = 0$
d) $r^3 - 6r^2 + 11r + 6 = 0$
e) None of the other answers is Correct.

Question 5: [2 points] Solving Linear Recurrence Relations
What is the solution of the recurrence relation $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$.

a) $a_n = 2 \cdot 3^n$
b) $a_n = 2 \cdot 2^n$
c) $a_n = 3 \cdot 2^n$
d) $a_n = 3 \cdot 3^n$
e) None of the other answers is Correct.

Question 6: [2 points] Propositional Logic
What is the contrapositive statement of the statement:
If it rained last night, then the sidewalk is wet.

a) If it did not rain last night, then the sidewalk is not wet.
b) If the sidewalk is wet, then it rained last night.
c) If it did not rain last night, then the sidewalk is wet.
d) If the sidewalk is not wet, then it did not rain last night.
e) None of the other answers is correct.

Question 7: [2 points] Probability Theory
The random variable X on a sample space S = {2, 4, 5, 6} has the following distribution:

$\boldsymbol{X}$	2	4	5	6
$\boldsymbol{P(X)}$	0.1	?	0.2	0.3

What is $P(X = 4)$ and what is P(X = 17)?

a) $P(X = 4) = 0.4$ and $P(X = 17) = 0.0$
b) $P(X = 4) = 4/17$ and $P(X = 17) = 0.0$
c) $P(X = 4) = 0.4$ and $P(X = 17) = 17/17$
d) $P(X = 4) = 1 - 0.4$ and $P(X = 17) = 0.4$
e) None of the other answers is Correct.

Question 8: [2 points] Recurrence Relations
What is the recurrence relation that has the characteristic equation
$r^2 - 6r + 9 = 0$?

a) $a_n = 6a_{n-1} + 9a_{n-2}$
b) $a_n = 6a_{n-1} - 9a_{n-2}$
c) $a_n = -6a_{n-1} + 9a_{n-2}$
d) $a_n = -6a_{n-1} - 9a_{n-2}$
e) None of the other answers is Correct.

Question 9: [2 points] Basic Structures – Functions
The sets A = {1, 2, 3, 4, 5} and B = {2, 3, 4, 5, 6}. Let f be a function from A to B, whose graph is {(1, 2), (2, 4), (3, 3), (4, 6), (5, 5)}. Choose the correct answer. The range of f is:

a) {1, 2, 3, 4, 5}
b) {3, 4, 5, 6}
c) {2, 3, 4, 5, 6}
d) {1, 2, 3, 4, 5, 6}
e) None of the other answers is correct.

Question 10: [2 points] Counting
What is the minimum number of students, each of whom comes from one of the 13 regions of Saud Arabia, who must be enrolled in KFUPM to guarantee that there are at least 50 who come from the same region?

a) 49 x 13 + 1
b) 49 x 13
c) 50 x 13 + 1
d) 50 x 13
e) None of the other answers is Correct.

Question 11: [2 points] Probability Theory
A bit string of length four is generated at random so that each of the 16-bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 1s, given that its first bit is a 1? (We assume that 0-bits and 1-bits are equally likely.)

a) $P(E|F) = \frac{1}{2}$
b) $P(E|F) = \frac{5}{8}$
c) $P(E|F) = \frac{1/16}{5/16}$
d) $P(E|F) = \frac{1}{4}$
e) None of the other answers is Correct.

Question 12: [2 points] Recursive Definition and Structural Induction
Give a recursive definition of the set of positive integers not divisible by 11.

a) 11 ∈ S, and if x ∈ S, then x + 11 ∈ S.
b) 11 ∈ S, and if x ∈ S, then 2x + 11 ∈ S.
c) 1 ∈ S, 2 ∈ S, 3 ∈ S, 4 ∈ S, 5 ∈ S, 6 ∈ S, 7 ∈ S, 8 ∈ S, 9 ∈ S, 10 ∈ S, and if x ∈ S, then 2x + 11 ∈ S.
d) There is no recursive definition for such set.
e) 1 ∈ S, 2 ∈ S, 3 ∈ S, 4 ∈ S, 5 ∈ S, 6 ∈ S, 7 ∈ S, 8 ∈ S, 9 ∈ S, 10 ∈ S, and if x ∈ S, then x + 11 ∈ S.

Question 13: [2 points] Probability Theory
Suppose *E* is an event with a probability of 0.9, and *F* is another event with a probability of 0.5. The probability of the intersection of E and F is 0.45. Are E and F independent events?

a) *Yes. E and F are independent events.*
b) *No. E and F are dependent events.*
c) *E is independent from F but F depends on E.*
d) *F is independent from E but E depends on F.*
e) None of the other answers is Correct.

Question 14: [2 points] Basic Structures – Functions
Choose the correct answer. Suppose f: N → N has the rule $f(n) = 3n^2 - 1$. Then,

a) *f* is onto but not one to one.
b) *f* is neither one to one nor onto.
c) *f* is one to one and onto.
d) *f is not a function.*
e) *f is one to one but not onto.*

Question 15: [2 points] Propositional Logic
The compound statement $\neg p \rightarrow (q \rightarrow r)$

a) Is a tautology
b) Is always False
c) Is False only when $p \equiv T, q \equiv T$ and $r \equiv T$
d) Is False when $p \equiv F, q \equiv T$ and $r \equiv F$
e) None of the other answers is correct

Question 16: [2 points] Cardinality of Sets
Given:

1. The positive integers less than 1,000,000,000 are Finite.
2. The integers that are multiples of 11 are countably infinite.
3. the real numbers between 0 and 0.5 are uncountable.
4. the real numbers between 2 and 10 are uncountable.

Select all correct statements from the above 4 statements.

a) None of the statement is correct.
b) Only 1 and 2 are correct.
c) Only 1 and 2 are correct.
d) Only 1 and 3 are correct.
e) All the statements are correct.

Question 17: [2 points] Binomial Coefficients and Identities
What is the coefficient of x^6y^{11} in the expansion of $(3x + 2y)^{17}$?

a) $\binom{17}{11} \times 3^6 \times 2^{11}$
b) $\binom{17}{6} \times 3^{11} \times 2^6$
c) $\binom{11}{6} \times 3^6 \times 2^{11}$
d) $\binom{11}{6} \times 3^{11} \times 2^6$
e) None of the other answers is Correct.

Question 18: [2 points] Probability Theory
What is the probability that in a group of 3 people chosen at random, there are at least two born in the same month of the year?

a) $1 - \frac{1}{12} * \frac{2}{12}$
b) $1 - \frac{1}{12} * \frac{2}{12} * \frac{3}{12}$
c) $1 - \frac{11}{12} * \frac{10}{12} * \frac{9}{12} = \frac{41}{96}$
d) $1 - \frac{11}{12} * \frac{10}{12}$
e) None of the other answers is Correct.

Question 19: [2 points] Basic Structures – Functions
The sets $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 4, 5, 6\}$. Let f be a function from A to B, whose graph is {(1, 2), (2, 4), (3, 3), (4, 6), (5, 5)}. The inverse, f^{-1} is:

a) {(1, 2), (2, 4), (3, 3), (4, 6), (5, 5)}
b) {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}
c) {(2, 1), (3, 3), (4, 2), (5, 5), (6, 4)}
d) {(1, 1), (2, 2), (4, 4), (3, 3), (4, 4), (5, 5)}
e) f has no inverse as it is not one to one.

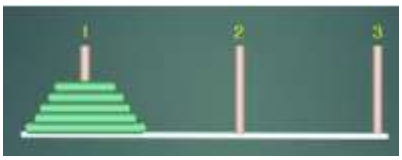
Question 20: [2 points] Basic Counting
How many strings of three decimal digits have exactly two digits that are 6s?

a) $\binom{3}{2} \times 1 \times 9 \times 9$
b) $\binom{3}{2} \times 1 \times 9$
c) $\binom{3}{2} \times 1 \times 10$
d) $\binom{3}{2} \times 1 \times 10 \times 10$
e) None of the other answers is Correct.

Question 21: [2 points] Predicates and Quantifiers
Suppose that variable x represents people, and that F(x) is "x is friendly," T(x) is "x is tall" and A(x) is "x is angry." The statement "Tall angry people are not friendly" is equivalent to

a) $\forall x\ (\neg F(x) \rightarrow (T(x) \wedge A(x)))$
b) $\exists x\ (T(x) \wedge A(x) \wedge \neg F(x))$
c) $\exists x\ ((T(x) \wedge A(x)) \rightarrow \neg F(x))$
d) $\forall x\ ((T(x) \wedge A(x)) \rightarrow \neg F(x))$
e) None of the other answers is correct.

Question 22: [2 points] Applications of Recurrence Relations



In Tower of Hanoi problem, the minimum number of moves needed to have all 5 disks moved from peg number 1 and stacked in the same order in peg number 3 is:

a) $3^5 - 1$.
b) $2^5 - 1$.
c) $5^2 - 1$.
d) $5^3 - 1$.
e) None of the other answers is Correct.

Question 23: [2 points] Probability Theory
A sequence of 8 bits is randomly generated. What is the probability that at least one of these bits is 0?

a) $\frac{1}{2^8}$
b) $\frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8} \times \frac{1}{2^8}$
c) $1 - \frac{1}{2^8}$
d) $\frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8}$
e) None of the other answers is Correct.

Question 24: [2 points] Propositional Logic
On the island of knights and knaves, you encounter two people: A and B. Person A says, "We are both knaves." Then,

a) A is a knight and B is a knight.
b) A is a knave and B is a knight.
c) A is a knight and B is a knave.
d) A is a knave and B is a knave.
e) One of them cannot be determined for sure whether he is knave or knight.

Question 25: [2 points] Basic Counting
A coin is flipped eight times where each flip comes up either head or tail. How many possible outcomes contain the same number of heads and tails? Assume a fare coin.

a) $\frac{8!}{4!}$
b) $4!$
c) $8!$
d) $\frac{8!}{4! \times 4!}$
e) None of the other answers is Correct.

Question 26: [2 points] Proposition Logic
What is the truth value of $(p \vee \neg q) \rightarrow q$ when $p \equiv F$ and $q \equiv F$?

a) True
b) False
c) Cannot be determined
d) The compound proposition is incorrect
e) None of the other answers is correct

Question 27: [2 points] Sets
The cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$

a) Is 2
b) Is 3
c) Is 4
d) Cannot be determined.
e) None of the other answers is correct

Question 28: [2 points] Quantifiers
The truth value of $\exists x \in \mathbb{R}(x^2 = x)$

a) Is True
b) Is False
c) True when $x = 1$ and False otherwise
d) Cannot be determined
e) None of the other answers is correct

Question 29: [2 points] Sets
How many different elements does A^n have when A has m elements and n is a positive integer?

a) n^m
b) m^n
c) $m + n$
d) $n \times m$
e) None of the other answers is correct

Question 30: [2 points] Probability Theory
Suppose E is the event that a randomly generated bit string of length five begins with a 1, and F is the event that this bit string contains an even number of 1s. Suppose all the 32-bit strings of length five are equally likely.
What is $p(E)$? What is $p(F)$? and What is $p(E \cap F)$?

a) $p(E) = 16/32 = ½$ $p(F) = 16/32 = ½$ $p(E \cap F) = 8/32 = ½$
b) $p(E) = 16/32 = ¼$ $p(F) = 16/32 = ¼$ $p(E \cap F) = 8/32 = ½$
c) $p(E) = 16/32 = ½$ $p(F) = 16/32 = ½$ $p(E \cap F) = 8/32 = ¼$
d) $p(E) = 16/32 = ¼$ $p(F) = 16/32 = ¼$ $p(E \cap F) = 8/32 = ¼$
e) None of the other answers is Correct.

Question 31: [2 points] Recursive Definition and Structural Induction
Let S be the subset of the set of ordered pairs of integers defined recursively by
Basis step: (0, 0) ∈ S.
Recursive step: If (a, b) ∈ S, then (a + 1, b + 2) ∈ S and (a + 2, b + 1) ∈ S.
The application of the basis step produces (0, 0) as an element in S.
Given the following sets of pairs:

1. (1, 2), (2, 1)
2. (2, 4), (3, 3), (4, 2)
3. (3, 6), (4, 5), (5, 4), (6, 3)
4. (4,8), (7,5), (6, 9)
5. (7,4), (8, 5), (9, 6)
6. (10, 7), (11, 8), (11, 7), (12, 8)

Select all elements of S produced by the next three applications of the recursive definition.

a) 1, 2, and 3 only.
b) 3 Only.
c) 1, ad 2 Only.
d) 1, 2, 3, 4, 5, and 6
e) None of the other answers is correct.

Question 32: [2 points] Propositional Logic
Suppose the domain of the propositional function R(w, z) consists of pairs w and z, where w is 7, or 9 and z is 3, 7, or 9. The proposition ∀w∃zR(w, z) can be written using disjunctions and conjunctions as

a) (R(7,3) ∨ R(7,7) ∨ R(7,9)) ∧ (R(9,3) ∨ R(9,7) ∨ R(9,9))
b) (R(7,3) ∧ R(7,7) ∧ R(7,9)) ∨ (R(9,3) ∧ R(9,7) ∧ R(9,9))
c) R(7,3) ∨ R(7,7) ∨ R(7,9) ∨ R(9,3) ∨ R(9,7) ∨ R(9,9)
d) R(7,3) ∧ R(7,7) ∧ R(7,9) ∧ R(9,3) ∧ R(9,7) ∧ R(9,9)
e) None of the other Answers is correct.

Question 33: [2 points] Recurrence Relations
Consider the recurrence relation $a_n = a_{n-1}^2$, which of the following fact is true

a) The recurrence relation is linear, homogeneous, of degree 1 and has constant coefficients.
b) The recurrence relation is linear, non-homogeneous of degree 2 and has constant coefficients.
c) The recurrence relation is linear, non-homogeneous of degree 1 and has constant coefficients.
d) The recurrence relation is non-linear, homogeneous of degree 1 and has constant coefficients.
e) None of the other answers is correct

Question 34: [2 points] Recurrence Relations
The recurrence relation for the number of bit strings of length *n* that do not have two consecutive 0s:

a) Is $a_n = 2^n - a_{n-2}$ for $n \geq 3$ and initial conditions $a_1 = 2$ and $a_2 = 3$
b) Is $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ and initial conditions $a_1 = 2$ and $a_2 = 3$
c) Is $a_n = 2a_{n-1} + 2a_{n-2}$ for $n \geq 3$ and initial conditions $a_1 = 1$ and $a_2 = 2$
d) Is $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$ and initial conditions $a_1 = 1$ and $a_2 = 2$
e) None of the other answers is correct

Question 35: [2 points] Propositional Logic
How many of the disjunctions $p \vee \neg q \vee s$, $\neg p \vee \neg r \vee s$, $\neg p \vee \neg r \vee \neg s$, $\neg p \vee q \vee \neg s$, $q \vee r \vee \neg s$, $q \vee \neg r \vee \neg s$, $\neg p \vee \neg q \vee \neg s$, $p \vee r \vee s$, and $p \vee r \vee \neg s$ can be made simultaneously true by an assignment of truth values to *p*, *q*, *r*, and *s*?

a) Only $p \vee \neg q \vee s$, $\neg p \vee \neg r \vee s$, $\neg p \vee \neg r \vee \neg s$, and $\neg p \vee q \vee \neg s$
b) Only $\neg p \vee \neg r \vee \neg s$, $\neg p \vee q \vee \neg s$, $q \vee \neg r \vee \neg s$, $\neg p \vee \neg q \vee \neg s$, $p \vee r \vee s$, and $p \vee r \vee \neg s$
c) None of them.
d) All of them.
e) None of the other answers is correct

ꕥ End of Questions ꕥ